

## **PRINCIPLE MULTIPLE OUTPUT BOTH SIDE GEAR WITH HARMONIC TRANSMISSION**

### **NAČELO VIŠESTRUKOG IZLAZA OBOSTRANOG OZUBLJENJA S HARMONIJSKIM PRIJENOSOM**

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**Ključne riječi:** harmonijski prijenos, mehanizam prijenosa, prijenosni omjer

**Key words:** harmonic transmission, transmission mechanism, transmission ratio

**Sažetak:** Rad prikazuje konstrukcijsko načelo višestrukog izlaza (odgona) obostranog diferencijalnih prijenosa s integriranim harmonijskim prijenosom. Prikazano načelo omogućava višestruke koaksijalne izlaze (odgone) s alternativnom izmjenom ulaza (pogona) i izlaza (odgona).

**Abstract:** The paper presents the design of the principle of multiple output both side differential transmissions with integrated harmonic transmission. The presented principle enables more coincidental coaxial outputs with the alternative of input and output interchange.

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## 1. INTRODUCTION

Department of technological equipment design of the FVT of the TU in Košice within the process of searching for alternatives to the so-called high precision transmissions of the constructions based on cycloidal teeth with the relatively complicated transmission of movement from satellite to centric has aimed its research at the new principles for design with relatively simple transmission of movement as well as presented transmissions regarding the number and range of transmission ratios.

The principle of integrated harmonic transmission with the multiple both side outputs has been suggested.

The paper also presents the kinematic analysis of the designed construction with transmission ratios for the certain levels.

## 2. PRINCIPLE OF MULTIPLE BOTH SIDE OUTPUT TRANSMISSIONS WITH INTEGRATED HARMONIC TRANSMISSION

Modern standard harmonic transmissions are usually designed as one-way with the only input and output without the possibility of their interchange.

The suggested multiple output transmission with integrated harmonic transmission is considered to be the new design with the variants of one input and three outputs with one braking of one of the transmission members. With the two levels of latitude, the transmission can operate as a differential. Output shafts are possible on both sides.

From the design principle it follows that alternative transmission from input shaft to the three output shafts (members) is achieved by means of one one-level transmission integrally interconnected with the two two-level transmissions within one construction of the whole transmission mechanism (fig. 1). Transmission of revolutions and performance for the suggested design is possible in two alternatives with the possible determination of corresponding transmission ratios.

### 2.1 The first alternative

Transmission of revolutions and performance is possible through the input eccentric shaft I, where fulcrum wheel 2 and 7 is located with gearing  $z_2$  and  $z_9$  and firmly connected gearing  $z_5$  and  $z_7$ , meshing with gearing  $z_6$  eventually  $z_8$  connected with the output shafts II, V.

The next output is possible either through the flexible member (disk wheel) 3 to the output shaft III, or through the crown wheel 4 connected with the output shaft IV. The second output is done through the flexible member 3 with the internal gear  $z_{3a}$  and external gear  $z_{3b}$ .

Thus, gearing  $z_2$  and  $z_9$  of the wheel 2 and 7 meshes with internal gear  $z_{3a}$  of the flexible member 3 and consequently the flexible member by means of external gear  $z_{3b}$  meshes with the internal gear  $z_4$  of the non-flexible member of the crown wheel 4. If the crown wheel 4 is braked, corresponding performance of the third output is transmitted to the shaft III by means of the flexible member 3. If the flexible member 3 is braked, performance is transmitted by means of the member 4 to the shaft IV.

### 2.2 The second alternative

To transmit revolutions and performance, the driveshaft II can be used. Then shafts I, III, V can be the input members, if the member 4 eventually I, IV, V is braked, if the member 3 is braked. Analogically it is possible to determine output shafts, if shaft V is a driveshaft.

Fig. 2 shows kinematic ratios between members 2 and 3 and consequently transmission

ratios for the first and the second alternatives have been derived.

### **3. TRANSMISSION RATIOS FOR THE FIRST ALTERNATIVE – SHAFT I IS A DRIVESHAFT**

### 3.1 Transmission ratios when member 3 is braked, $n_3 = 0$

- a) Transmission ratio between shaft I and gear wheel 2 if the flexible member 3 is braked. The analogue relations are also between shaft I and gear wheel 7. From the kinematic ratios shown in fig. 2 it follows that for the velocity vector at point A the following is valid:

$$v_{A20} = v_{A30} \quad (1)$$

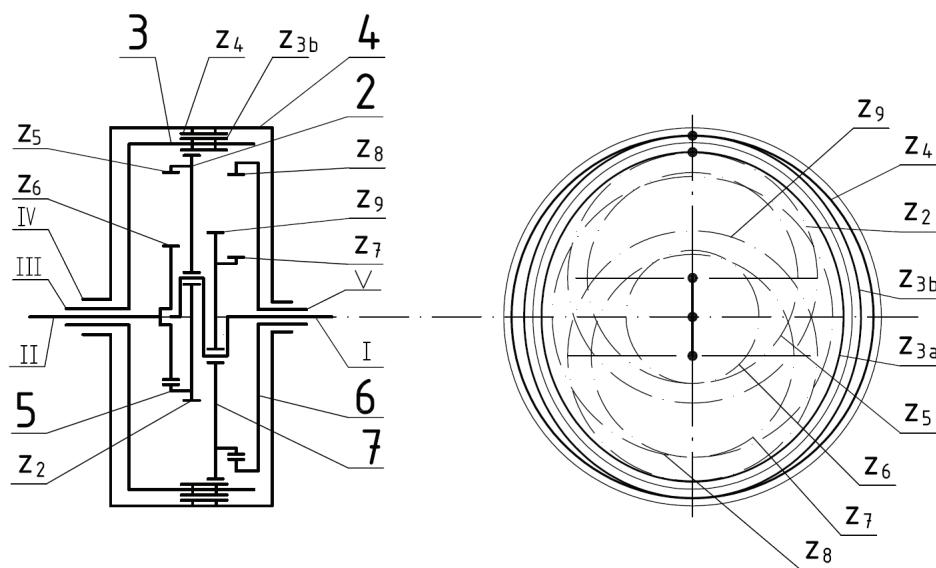


Fig. 1 Principle of integrated harmonic transmission mechanism

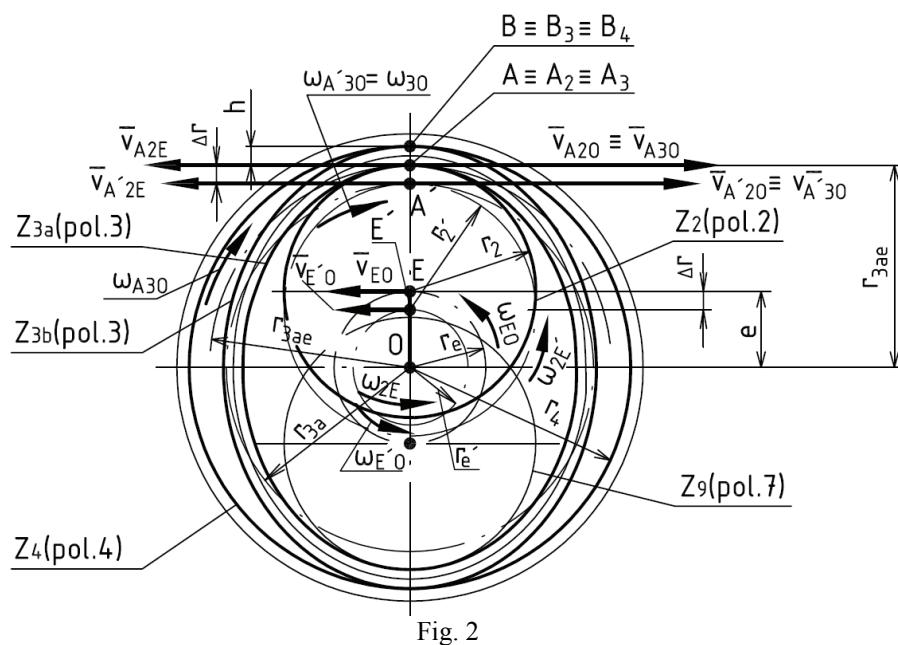


Fig. 2

At point A the pitch circle of the gearing  $z_2$  of the member 2 touches the pitch ellipse of gearing  $z_{3a}$  of the deformed member 3, basic shape of which is circle with pitch circle radius  $r_{3a}$ , of the gearing  $z_{3a}$ . At the same time, meshing of gearing  $z_4$  of the member 4 with gearing  $z_{3b}$  of the member 3 at point B occurs. At this point the pitch circle touches gearing  $z_4$  with the pitch ellipse of the gearing  $z_{3b}$  of the deformed member 3. It follows that big (main) axle of pitch ellipse of gearing  $z_{3a}$  at the point A has the size  $r_{3ae}$ . It follows from the above as well as from fig. 2 that:  $r_{3ae} = r_e + r_2$   $r_{3ae} = r_{3af}$

where:

$r_e$  - axle eccentricity of the member 2 rotation

$r_2$  - radius of pitch circle of gearing  $z_2$  of the member 2.

Radius  $r_{3ae}$  is smaller than  $r_4$  by a value  $h$  (see fig. 1).

Thus:  $h = r_4 - r_{3ae}$

Value  $h$  usually follows from the body 3 construction based on pre-determined requirements for a transmission. From the vector kinematics for velocity at the point A<sub>2</sub> regarding point O the following is valid:

$$\bar{v}_{A2O} = \bar{v}_{A2E} + \bar{v}_{EO} = \bar{v}_{A3O} \quad (2)$$

From the equation (1) it follows that at the point A the following is valid:

$$\omega_{A2O} = \omega_{A3O} \quad (3)$$

After substitution the velocity vectors at the point A

$$\bar{v}_{A3O} = \omega_{A3O} \cdot r_{3ae}, \quad \bar{v}_{A2E} = \omega_{2E} \cdot r_2, \quad \bar{v}_{EO} = \omega_{EO} \cdot r_e$$

and if we take into consideration the supposed direction of revolution regarding fig. 2 the following is valid:

$$\omega_{A3O} \cdot r_{3ae} = \omega_{2E} \cdot r_2 + \omega_{EO} \cdot r_e \quad (4)$$

Regarding the fact that body 3 is in the part of gearing  $z_{3a}$ ,  $z_{3b}$  flexible,  $\omega_{A3O}$  it does not always equal the angle velocity of the body 3, thus  $\omega_{A3O} \neq \omega_{3O}$ . Angle velocity of the non-deformed body 3 equals angle velocity of the input shaft III. With constant  $\omega_I = \omega_{EO}$  also  $\omega_{3O} = \text{constant}$ . When deriving the transmission ratio  $\omega_{3O}$  between shaft I and wheel 2 it is necessary to follow from circumferential velocity  $v_{A'3O}$  at point A' on the pitch circle of gearing  $z_{3a}$  of the non-deformed member 3. Thus  $v_{A'3O} = \omega_{A'3O} r_{3a}$ , with  $\omega_{A'3O} = \omega_{3O}$ .

If we consider that body 3 (flexible member) is braked in the body of transmission then when deriving the transmission ratio between input shaft and wheel 2 we follow form the fact that gear wheel 2 when shaft I with the certain slew angle performs the same mesh trajectory of deformed gear member 3 by gearing  $z_{3a}$  as in case of non-deformed member 3 with the other corresponding eccentricity  $r_{e'}$ , that is smaller than eccentricity  $r_e$  by a value of deformation  $\Delta r$  (see fig. 2). Based on the above mentioned, it is possible to write the following for the velocity vector at the point A', which represents the meshing the gear wheel 2 with non-deformed member 3 with the gearing  $z_{3a}$ :

$$\bar{v}_{A'2O} = \bar{v}_{A'2E} + \bar{v}_{E'o} = \bar{v}_{A'3O} \quad (5)$$

$$\text{after substitution: } \omega_{A'3O} \cdot r_{3a} = \omega_{2E} \cdot r_2 + \omega_{E'O} \cdot r_{e'} \quad (6)$$

and the following is valid:  $r_{e'} = r_{3a} - r_2$ ,  $\omega_{2E} = \omega_2$ ,  $\omega_{E'O} = \omega_{EO} = \omega_1$

after substitution the following is valid:

$$-\frac{\omega_I}{\omega_2} = \frac{r_2}{r_{3a} - r_2} \quad (6)$$

Expression  $\frac{\omega_I}{\omega_2}$  is the searched transmission ratio. Let us denote it by  $u_{I-2}^3$ , then we can write:

$$u_{I-2}^3 = -\frac{r_2}{r_{3a} - r_2} \text{ or } u_{I-2}^3 = \frac{r_2}{r_2 - r_{3a}} \quad (7)$$

where  $u_{I-2}^3$  - is transmission ratio between shaft I and wheel 2 when member 3 is braked.

If we substitute  $r_{3a} = \frac{mz_{3a}}{2}$  and  $r_2 = \frac{mz_2}{2}$ , then transmission ratio  $u_{I-2}^3$  is as follows:

$$u_{I-2}^3 = \frac{z_2}{z_2 - z_{3a}} = \frac{\omega_I}{\omega_2} \quad (8)$$

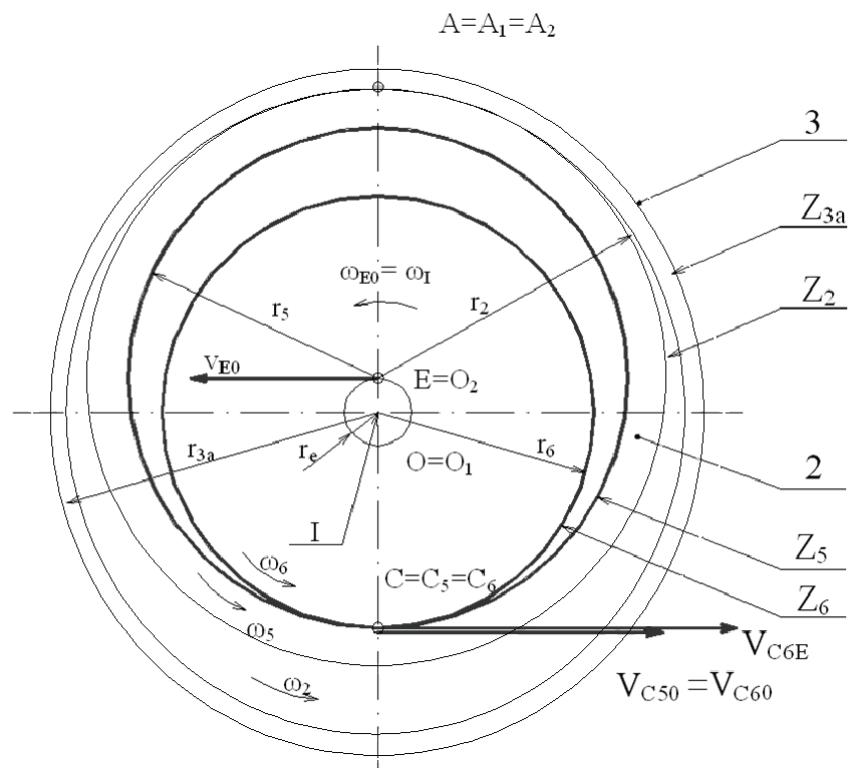


Fig. 3 Kinematic scheme of the second level transmission.  
 Output shaft with the external gear wheel.

b) Transmission ratio between the shafts I and IV when member 3 is braked. Output shaft V has internal gear wheel. From the equation (6) and (8) for kinematic ratios of gear wheels  $z_2$  and  $z_3$  of the harmonic transmission for angle velocity of the internal gear wheel 2 it follows

$$\omega_2 = \omega_I \frac{r_1 - r_2}{r_2} \quad (9)$$

If annual gears  $z_2$  and  $z_8$  are on the same body of the gear wheel the following is valid:  $\omega_2 = \omega_8$ . Eccentricity  $e$  at the same time equals the radius of revolution of the wheel axis 2 (of the point E) around the point O of the central axis of the transmission.

Thus:  $e = r_e = r_6 - r_5 = r_{3a} - r_2$ . (10)

From fig. 3 it follows:  $\bar{v}_{C6O} = \bar{v}_{6E} + \bar{v}_{EO}$ , (11)

At the same time the following is valid:

$v_{C6O} = v_{C5O}$ , then  $v_{C6O} = r_6 \omega_6$ ,  $v_{C6E} = r_3 \omega_5$ ,  $v_{EO} = r_e \omega_{EO}$ , and  $\omega_{EO} = \omega_I$ .

After substitution and regarding the selected revolution according to fig. 3 we will obtain:

$$r_6 \omega_6 = r_5 \omega_5 + r_e \omega_I \quad (12)$$

If  $\omega_5 = \omega_2$  (gearings  $z_5$  and  $z_2$  are firmly gripped on the wheel 2) then we can substitute from the equation (9)  $\omega_2 = \omega_5$  to the equation (12). From the transmission design it follows that angle velocity of the shaft V equals the angle velocity of the gear wheel  $z_4$ , because of the firm connection with the shaft V. Thus  $\omega_6 = \omega_V$ . After substitution we will obtain:

$$r_6 \omega_V = r_5 \left( \omega_I \frac{r_2 - r_{3a}}{r_2} \right) + (r_6 - r_5) \omega_I$$

Ratio  $\frac{\omega_I}{\omega_V}$  is the transmission ratio between shaft I and V when member 3 is braked. Let us denote it by  $u_{I-V}^3$  and angle velocity  $\omega_V$  by  $\omega_V^3$  (index 3 – braked member 3, and consequently we will obtain the following:  $u_{I-V}^3 = \frac{\omega_I}{\omega_V^3} = \frac{r_6 r_2}{r_5 (r_2 - r_{3a}) + r_2 (r_6 - r_5)}$  (13)

After the radius through teeth number has been defined:

$$u_{I-V}^3 = \frac{\omega_I}{\omega_V^3} = \frac{z_6 z_2}{z_5 (z_2 - z_{3a}) + z_2 (z_6 - z_5)} \quad (14)$$

Transmission ratios between the shaft I and II with the corresponding numbers of teeth for gearing are of the analogical form.

c) Transmission ratio between the shaft I and IV when member 3 is braked. For this transmission, analogical relationship known from harmonic transmissions can be derived. Thus, the following is valid:

$$u_{I-IV}^3 = \frac{z_4}{z_4 - z_{3b}} \quad (15)$$

### 3.2 Transmission ratios when member 4 is braked and $n_4 = 0$ .

a) Transmission ratio between the shaft I and III. For the transmission ratio the following relationship can be derived:

$$u_{I-III}^4 = \frac{\omega_I}{\omega_{III}^4} = \frac{z_{3b}}{z_{3b} - z_4} \quad (16)$$

where  $\omega_{III}^4$  - is the angle velocity of the shaft III when member 4 is braked.

b) Transmission ratio between the shaft I and V. For the angle velocity of the shaft V the following is valid:

$$\omega_V^4 = \omega_V^3 + \omega_{III}^4 \quad (17)$$

where

$\omega_V^4$  - is the angle velocity of the shaft V when member 4 is braked

$\omega_V^3$  - is the angle velocity of the shaft V when member 3 is braked

Let us denote transmission ratio between the shaft I and V when member 4 is braked by  $u_{I-V}^4$ , then

$$u_{I-V}^4 = \frac{\omega_I}{\omega_V^4} = \frac{\omega_I}{\omega_V^3 + \omega_{III}^4} \quad (18)$$

After expression  $\omega_V^3$  from the equation (14) and  $\omega_{III}^4$  from the equation (16) and after substitution to the equation (18) we will obtain:

$$u_{I-V}^4 = \frac{z_6 z_2 z_{3b}}{[z_5(z_2 - z_{3a}) + z_2(z_6 - z_5)]z_{3b} + (z_{3b} - z_4)z_6 z_2}$$

Relationship for transmission ratio  $u_{I-II}^4$  between the shafts I and II when member 4 is braked is of the analogical form, although when the corresponding number of teeth of the gearing are substituted.

## 4. TRANSMISSION RATIOS FOR THE SECOND ALTERNATIVE – SHAFT II IS THE DRIVESHAFT

### 4.1 If the wheel 4 is braked and $n_4 = 0$ .

- a) Transmission ratio between the shafts II and I:  $u_{II-I}^4 = \frac{1}{u_{I-II}^4}$
- b) Transmission ratio between the shafts II and III:  $u_{II-III}^4 = \frac{u_{I-III}^4}{u_{I-II}^4}$

c) Transmission ratio between the shafts II and V:  $u_{II-V}^4 = \frac{u_{I-V}^4}{u_{I-II}^4}$

#### 4.2 Transmission ratios when member 3 is braked and $n_3 = 0$

- a) Transmission ratio between the shafts II and I:  $u_{II-I}^3 = \frac{1}{u_{I-II}^3}$
- b) Transmission ratio between the shafts II and IV:  $u_{II-IV}^3 = \frac{u_{I-IV}^3}{u_{I-II}^3}$
- c) Transmission ratio between the shafts II and V:  $u_{II-V}^3 = \frac{u_{I-V}^3}{u_{I-II}^3}$

### 5. CONCLUSION

The objective of the paper is to present principally new design of multiple output both side transmission with the possibility of input and output interchange with the alternatives of the possible three outputs from the given transmission mechanism. Based on the kinematic scheme, corresponding transmission ratios have been derived. Within the further research, force ratios analysis and possible loadings of the mentioned mechanism, eventually the other corresponding solutions [1], [4], [9] will be given great attention. This contribution forms a part of the solution of the grant task VEGA 1/4156/07 and KEGA 3/6279/08.

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