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MELTING OF GALLIUM WITH THE INFLUENCE OF NATURAL CONVECTION

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Abstract

In this paper, the finite volume method was applied to solve the mathematical model of melting of pure substance with the influence of natural convection in a two-dimensional situation. In the mathematical model, laminar fluid flow was assumed, and buoyancy forces were modeled using the Boussinesq approximation. The main features of the numerical method used are the application of a fixed Cartesian geometric grid with simultaneous solution of the energy equation in the solid and liquid phases, where the boundary condition at the phase boundary is satisfied by the source term of the energy equation. The developed method was applied to the problem of melting of gallium. By simulating the experimental situation in the melting of gallium, relatively good agreement was obtained with the measurement results. The influence of natural convection on the melting process is important.

Keywords: Melting, Gallium, Natural convection, Finite volume method, SIMPLER

1. Introduction

Melting and solidification as solid/liquid phase change phenomena are common in nature. Solidification and convection of lava influence tectonic activity, and melting and freezing of glaciers are critical for climate stability. Understanding these processes makes it possible to predict natural phenomena and improve living standards. Today, phase change phenomena between solid and liquid are an required part of many technological processes, e.g., in the production of glass, crystals, and metal alloys; in continuous casting; in welding; in food preservation; in seawater desalination; in the production of ice and ice cream; and they are key mechanisms in refrigeration and air conditioning systems. Recently, researchers' interest in these phenomena has increased due to the possibility of their application in latent heat storage systems, especially for the efficient storage of solar energy [1]. In the processes of melting and solidification, the substance changes its state of aggregation from

solid to liquid or from liquid to solid. This change occurs at a specific temperature for pure substances and eutectics and over a range of temperatures for binary and multicomponent alloys. Among the numerous works dealing with phase changes of multicomponent mixtures, the following works can be highlighted [2,3]. In the phase change of pure substances, which is the subject of this work, the solid and liquid phases with different thermophysical properties are separated by a phase boundary of negligible thickness, at which heat is released or absorbed. Mathematically, melting and solidification processes belong to the group of problems with a moving and free boundary. The main feature of this group of problems is the existence of a phase boundary that depends on time and space, whose position cannot be determined in advance, but is an integral part of the solution. The being of a moving boundary makes these problems nonlinear and is the main cause of the complexity of their solution. Numerous numerical methods from both groups of methods, finite element and finite volume/difference methods, have been and are being developed to solve phase change problems. Numerical methods for solving moving boundary problems can be classified into two basic groups: Variable or dynamic grid methods [4] and fixed grid methods [5]. From the numerous literature dealing with the numerical solution of phase change problems, articles can be singled out in which two basic groups of numerical methods are systematically compared [6]. The analysis of the melting/solidification process aims at predicting the time change of the temperature field in the solid and liquid phases, the position of the phase boundary and the rate of phase transformation, determining the velocity and pressure field in the liquid phase, and determining the amount of heat exchanged. The paper uses a method for calculating the required fields during the phase change, which includes the establishment of a mathematical model and the development and implementation of a finite volume method for simultaneously solving the equations in the solid and liquid phases while maintaining certain conditions at the time-varying boundary between these phases.

2. Mathematical model and numerical method

The governing equations for unsteady two-dimensional melting with natural convection are the continuity, momentum, and energy equations. They are subject to certain assumptions, including that the liquid phase is incompressible and satisfies the Boussinesq approximation, that the flow is laminar, and that viscous dissipation can be neglected. The mathematical model is expressed in dimensionless form, [7]

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \qquad (1)$$

$$\frac{\partial U}{\partial \tau}+U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-\frac{\partial P'}{\partial X}+Pr\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right) \qquad (2)$$

$$\frac{\partial V}{\partial \tau}+U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=GrPr^2\left(\theta-\theta_0\right)-\frac{\partial P'}{\partial Y}+Pr\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right) \qquad (3)$$

$$\frac{\partial \theta}{\partial \tau}+\left(U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}\right)=\left(\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2}\right)+\frac{1}{Ste}\cdot\frac{\partial f}{\partial \tau} \qquad (4)$$

The problem under consideration is governed by the boundary conditions at the solid, impermeable, isothermal, or insulated wall and at the solid-liquid interface. At the impermeable wall, both velocity components are set to zero. At the isothermal wall, the temperature is prescribed, while at the insulated wall, the normal derivative of the temperature is zero. At the solid-liquid interface, the temperature is constant and equal to the melting temperature T_m. If the interface velocity is negligible, the Stefan condition at the interface follows from the energy equation, in the form

$$k_s\left.\frac{\partial T}{\partial n}\right|_s - k_l\left.\frac{\partial T}{\partial n}\right|_l = \rho_s h_L \frac{\partial n}{\partial t} \qquad (5)$$

where the indices l and s refer to the liquid and solid phase, respectively, and n is the coordinate normal to the phase boundary. The phase change problem can be specified by three dimensionless parameters: Rayleigh number, Prandtl number and Stefan number. To solve the equations of the mathematical model, the finite volume method with a fixed rectangular grid was used. The governing equations are discretized using the Exponential differencing scheme for the spatial derivatives and the fully implicit scheme for the time integration. The SIMPLER algorithm is used for the pressure-velocity coupling. Numerical simulations of gallium melting were performed using the FORTRAN computer code. Figure 1 shows a vertical cross-section of the rectangular cavity of dimensions W and H containing gallium, including boundary conditions. The northern and southern boundaries are adiabatic, while the eastern and western boundaries are isothermal. At the beginning of the simulation, the cavity is filled with a solid at initial temperature T_i equal to or less than the melting temperature ($T_i \leq T_m$). At $t = 0$, the western vertical wall is heated to a temperature higher than the melting temperature ($T_W > T_m$), while the eastern wall is kept at the temperature T_E. The position of the interface at $t > 0$ is also shown in Figure 1.

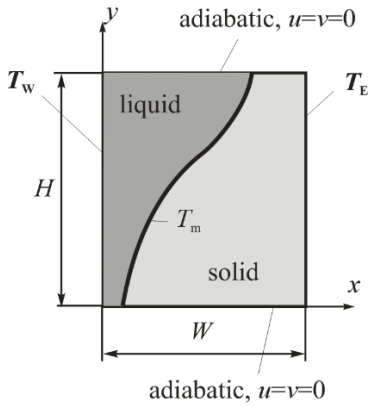

Figure 1. Schematic view of the melting process

3. Simulation of the experimental situation with melting of gallium

In this paper the developed method is applied to melting of gallium under the influence of natural convection and the results are compared with experimental results and numerical results of Beckermann, [8]. The substance is gallium (purity 99.99%), which is often used for experiments due to its low melting temperature (T_m=29,78 °C) and known physical properties. Table 1 shows the values of physical properties of gallium (solid and liquid phases) and other relevant data for the experiment performed. The temperature of the warm wall is T_W=39,98 °C, and the temperature of the cold wall is T_E=24,98 °C. The initial temperature of the solid is T_i=24,98 °C.

Table 1. Values of dimensionless quantities and physical properties of substances in the melting of gallium under the influence of natural convection

Ra	$3{,}275\cdot10^{5}$
Gr	$1{,}575\cdot10^{7}$
Pr	0,0208
Ste	0,0507
Sb	0,468
λ_s, W/(m·K)	38,37
λ_l, W/(m·K)	38,37
c_s, J/(kg·K)	368,93
c_l, J/(kg·K)	395
ρ_s, kg/m^3	5910
ρ_l, kg/m^3	6099
a_s, m^2/s	$1{,}76\cdot10^{-5}$
a_l, m^2/s	$1{,}59\cdot10^{-5}$
h_L, J/kg	$7{,}94\cdot10^{4}$
T_m, °C	29,78
ΔT_s, °C	4,8
ΔT_l, °C	10,2
β, K^{-1}	$1{,}6\cdot10^{-4}$
H=W, m	0,0476
B, m	1

Dimensionless numbers are defined as follows:

Rayleigh number: $Ra = \dfrac{g\beta\Delta T_l H^3}{\nu_l a_l}$

Prandtl number : $Pr = \dfrac{\nu_l}{a_l}$

Stefan number: $Ste = \dfrac{c_l \Delta T_l}{h_L}$

Subcooling parameter: $Sb = \dfrac{\Delta T_s}{\Delta T_l}$

Mean Nusselt number on the warm wall

$$\overline{Nu}_W = \frac{\Phi_W \cdot W}{\lambda_l \cdot \Delta T_l \cdot H \cdot B}$$

Mean Nusselt number on the cold wall

$$\overline{Nu}_E = \frac{\Phi_E \cdot W}{\lambda_l \cdot \Delta T_l \cdot H \cdot B}$$

Dimensionless time:

$$\tau = Fo \cdot Ste = a_l \cdot t \cdot Ste / H^2$$

Dimensionless temperatures

$$\theta_l = \frac{T - T_m}{\Delta T_l} \qquad \theta_s = \frac{T - T_m}{\Delta T_s}$$

The experiment was conducted in a well-insulated test cavity of square cross-section. The test cavity had inside dimensions of 4.76 cm in height and width and 3.81 cm in depth. The price of gallium is relatively high, so a small test cavity was chosen. The temperature distribution in the cavity was measured with 33 thermocouples placed at three heights ($Y=y/H$ =0.133; 0.5; 0.867). During melting, gallium contracts (volume decreases about 3%), so it is necessary to allow gallium to enter the cavity through small holes in the top plate.

The diameter of the thermocouple wire was 0.127 mm, and the thermocouple holders were placed in such a way that the flow caused by natural convection was disturbed as little as possible. The uncertainty in the location of each thermocouple bead was approximately ±2 mm. All thermocouples were calibrated with accuracy of ±0.1 ◦C. In addition to the measurement results, the authors of the article [8] have also included the calculation results obtained using the method with two geometric grids of 26x26 volume in the liquid phase and the same volume in the solid phase. The calculation was performed for a two dimensional situation. The calculated temperature values agree with the measured values within 5% of the total wall temperature difference. Discrepancies can mainly be explained by the unreliability of the experiment, of which the inaccuracy in determining the position of the thermocouples is the biggest reason, as well as the impossibility of fully achieving isothermal and adiabatic boundary conditions.

The unreliability of the numerical calculation resulted mainly from the neglect of the three-dimensional structure of natural convection. Smaller inaccuracies can be caused by neglecting the inflow of gallium due to contraction and the change of thermophysical properties with temperature.

The numerical results obtained in this work are compared with experimental measurements and numerical results from the article [8].

The calculation was performed on two uniform geometric grids M1 31x31 control volume and M2 60x60 control volume. Since there is little difference between the results obtained on these grids, only the results of the M2 grid are presented in this article. The total integration time is 3600 seconds, and the integration time step is 0.1 s.

Figure 2 shows the image of isotherms at three time points t_1=3 min, t_2=10 min, t_3=50 min, where figures a1), a2) and a3) show the results obtained in this work and figures b1), b2) and b3) represent the results of the calculations of Beckermann and Viskanta.

Figure 3 shows streamlines. Figures a1), a2), and a3) again show the results obtained in this work and refer to time points t_1=3 min, t_2=10 min, t_3=50 min and figures b1), b2) and b3) show the results of the calculations of Beckermann and Viskanta.

Figure 4 shows the change in the mean values of the Nusselt number on the warm and cold walls.

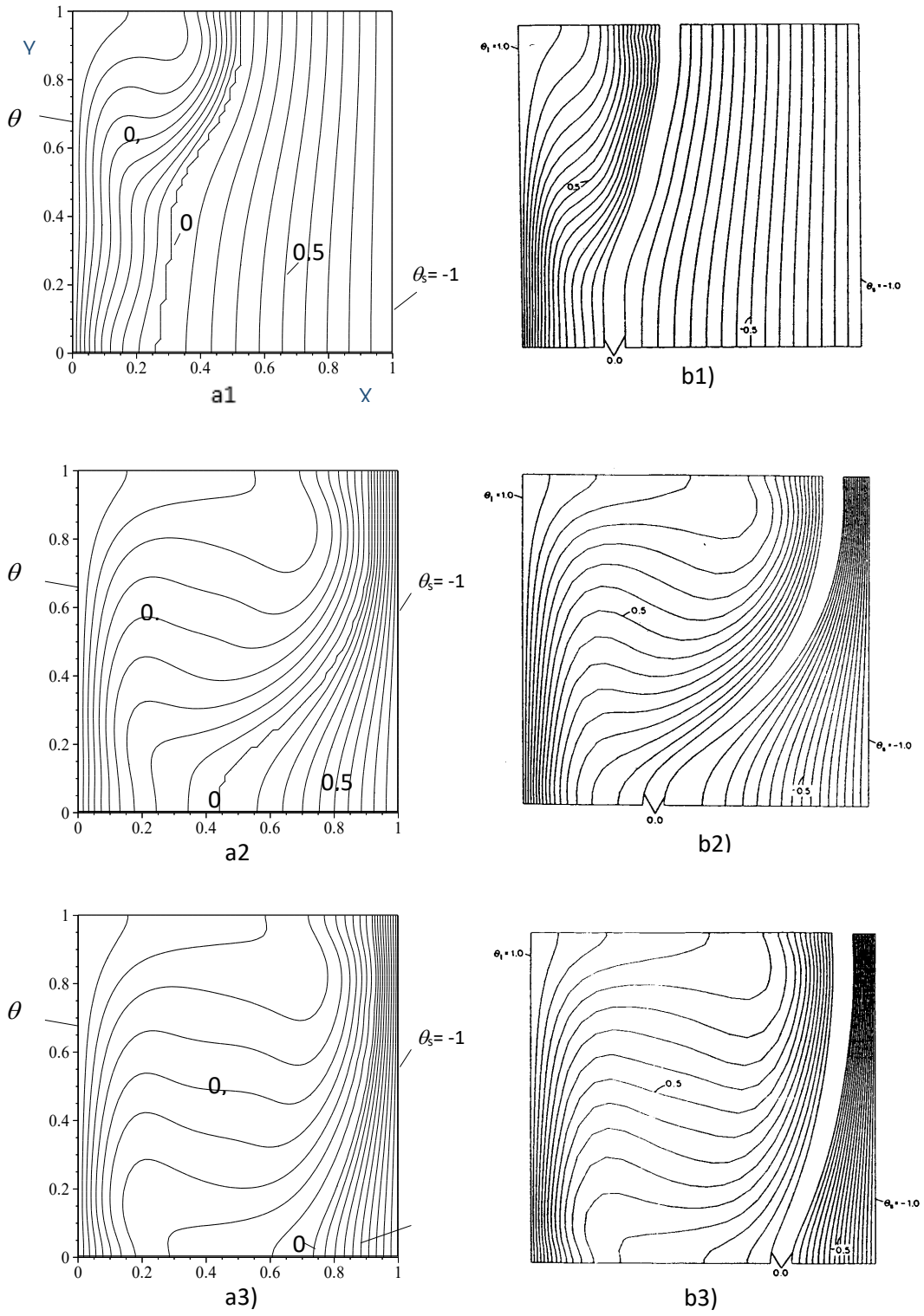

Figure 2. Isotherms: comparison of numerical results: a) this work, b) Beckermann

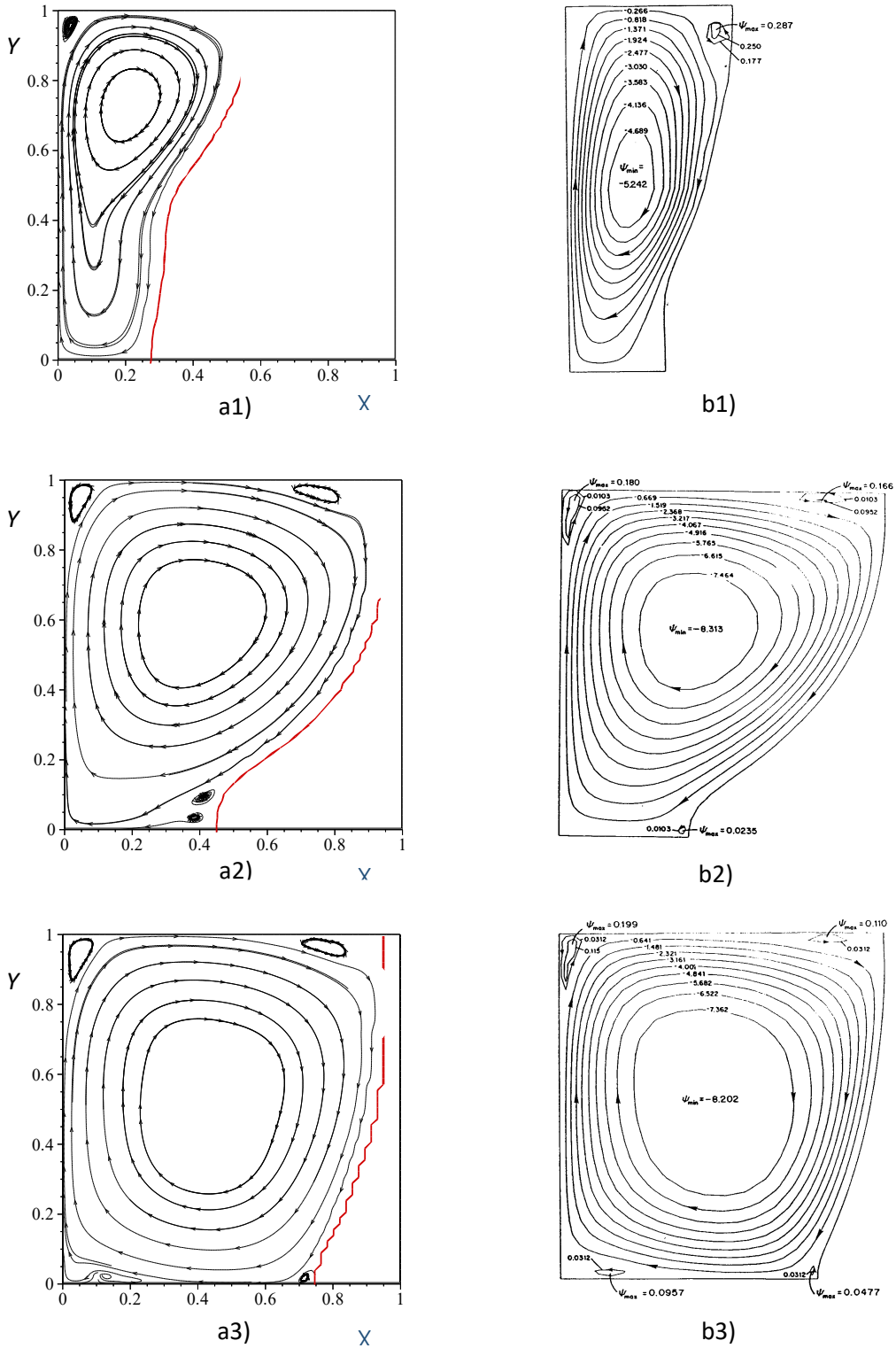

Figure 3. Streamlines: comparison of numerical results: a) this work, b) Beckermann

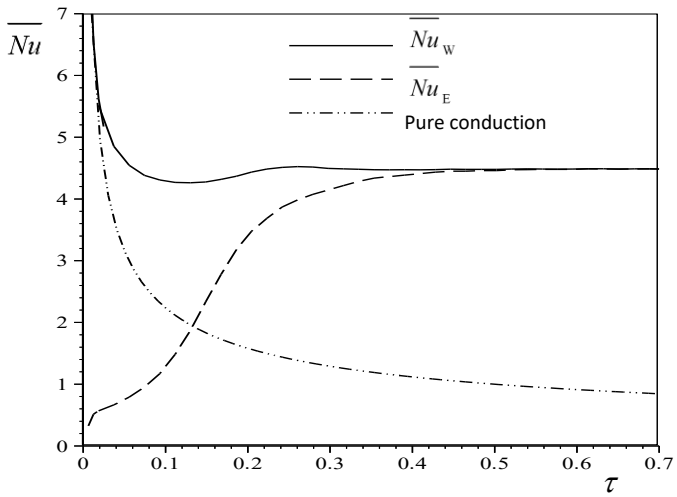

Figure 4. Temporal change of the mean value of the Nusselt number on the warm wall ($\overline{Nu}_w$) and mean value of the Nusselt number on the cold wall ($\overline{Nu}_E$) in the numerical simulation of the melting of gallium

4. Discussion of the results

The melting process consists of four regimes

- A pure conduction regime, in which the interface moves vertically
- A regime with the initial development of convection, where the convection flow occurs in the upper part of the cavity, where the interface is further away from the vertical wall, and a conduction regime in the lower part of the cavity, where the interface remains vertical
- The regime of developed convection, where the interface is curved and moves faster in the upper part of the cavity than in the lower part of the cavity
- The regime where melting takes place in the lower part of the cavity and the interface reaches a steady state

Figures 2a1), 2b1) and 3a1), 3b1) show typical isotherms and streamlines for the second regime at *t*=3 min. In the upper part of the melt, natural convection takes place, while thermal boundary layers are present on the warm wall and along the phase boundary. In the lower part, conduction is still present, which means that the interface is vertical. Weak recirculation cells are observed in the upper right part of the melt, which is typical of convection in fluids with a low Prandtl number. The second regime ends shortly after *t*=3 min, when the convection region has spread to the bottom of the enclosure. Figure 4 shows that at this time, the average Nusselt number at the hot wall reaches a local minimum. The values of the Nusselt number at the cold wall are low, which means that the heat flux

dissipated by the cold wall has low values. Subcooling of the solid in the first two regimes has a little effect on the melting process.

During the third regime, natural convection is present throughout the molten region. The interface is curved, and the temperature gradients and melting rate are higher in the upper part than in the lower part of the cavity. The interface is at the upper and lower adiabatic boundary perpendicular to the top and bottom plates. Figures 2a2), 2b2) and 3a2), 3b2) show the predicted isotherms and streamlines at t=10 min just before the end of the third regime.

The distinctive feature of this regime is the appearance of new recirculation cells. They are now observed in the upper and lower right corners of the molten region. There are thermal boundary layers along the interface and on the hot wall. In the upper third of the cavity, the interface has almost approached the cold wall, so that the temperature gradients in the solid are large in this part of the cavity. The interface is strongly curved, and in the lower part of the cavity the width of the solid is still large, and here the temperature gradients are relatively small. The Nusselt number at the hot wall decreases slightly from a value of 4.8 at the beginning to 4.5 to the end of the third regime. On the other hand, the Nusselt number at the cold wall increases sharply, indicating that the melting process is progressively effected by heat conduction through the solid. The isotherms and streamlines at t=50 min are shown in figures 2a3), 2b3) and 3a3), 3b3). At the given time, the interface has reached a stationary position, and the heat transfer rates through the solid and the liquid are equal. The natural convection patterns in the liquid region bear a close resemblance to those of a vertical square cavity. As a result of the natural convection flowing downward along the interface, the interface still demonstrates a slight curvature that causes a variation in the local heat transfer rates. Figure 4 indicates that the Nusselt number at the hot wall remains approximately constant during the third and fourth regimes, while the Nusselt number at the cold wall continues to increase until it approaches the same value as the Nusselt number at the hot wall.

A comparison was made between the predicted and measured interface positions in the steady state. Predicted temperatures were found to agree with experimental results within a margin of approximately 5% of the total temperature difference in the test cell. It is believed that experimental uncertainties such as inaccuracies in thermocouple locations and reading, as well as imperfect boundary conditions can primarily be attributed to the discrepancies between the predicted and measured values.

Throughout the experiment, a lower intensity of natural convection was observed in the melt region than predicted in the measurements. This difference can be attributed to the presence of the thermocouple rake in the test cell, which may have disturbed the flow of the melt.

5. Conclusions

In the paper, the finite volume method was applied to solve the mathematical model of the melting of pure substances in free convection in a two-dimensional situation. In the mathematical model, laminar fluid flow was assumed, and buoyancy forces were modeled using the Boussinesq

approximation. The main features of the numerical method used are the application of a fixed Cartesian geometric grid with simultaneous solution of the energy equation in the solid and fluid phases, where the boundary condition at the phase boundary is satisfied by the original term of the energy equation. The developed method was applied to the problem of melting of gallium.
Analyzes of the influence of natural convection on melting processes in rectangular geometries, carried out experimentally and numerically and published in the literature, showed that heat transfer during the melting process occurs through regimes: pure conduction regime, transition regime of conduction and convection, and the dominant convection regime. The simulation of the experimental situation during the melting of gallium showed relatively good agreement with the measurement results, similar to the agreement of the simulation results obtained by the author of the experiment. It is shown that natural convection has a large influence on the melting of a gallium.

6. References

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